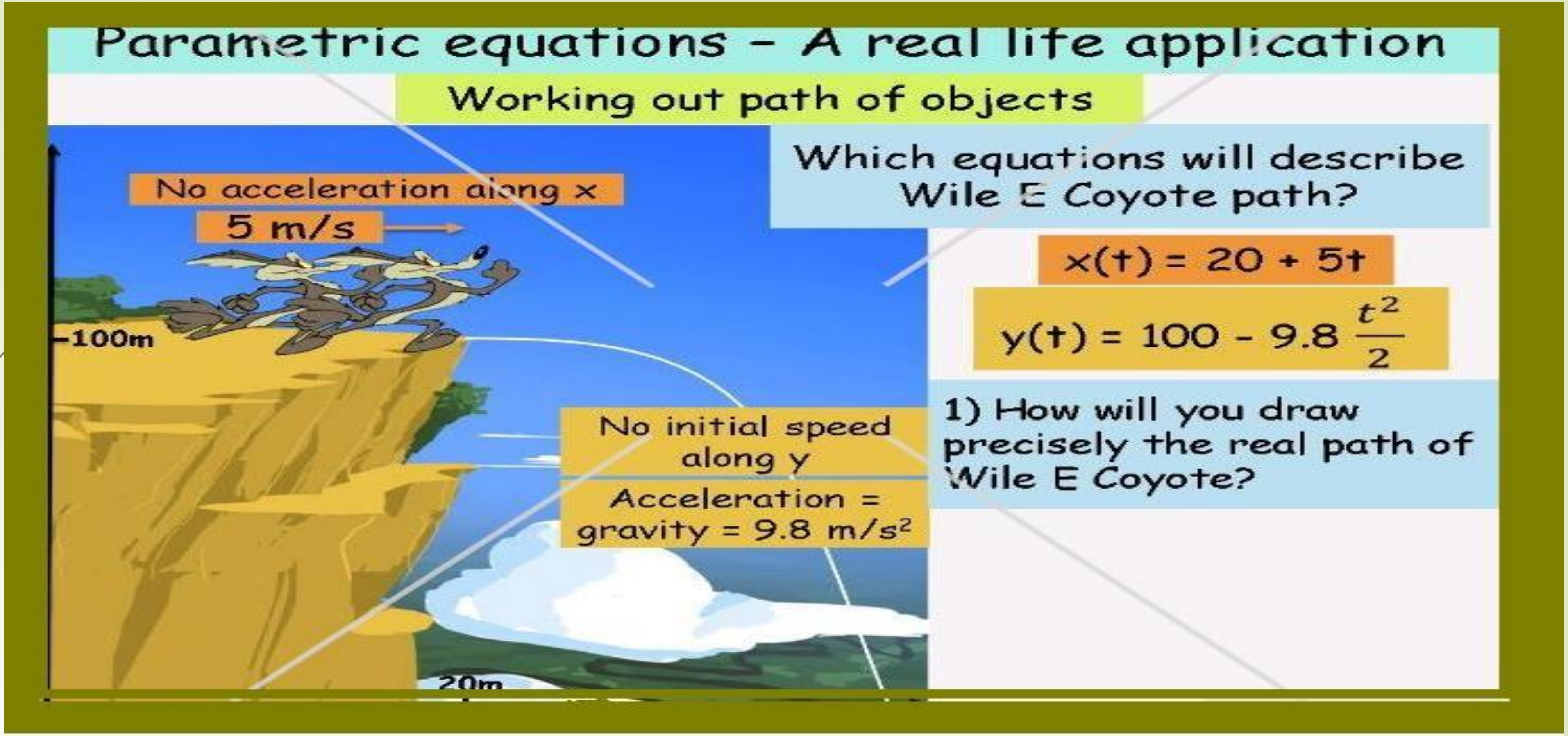
A REAL LIFE APPLICATION



t is the 3rd variable

$$y = 5t - t^2 \qquad x = 4t^4 + 2$$

$$\frac{dy}{dt} = 5 - 2t \qquad \frac{dx}{dt} = 16t^3$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{dy}{d} \div \frac{dx}{d}$$

$$\frac{dy}{dx} = \frac{5 - 2t}{16t^3}$$

t is the 3rd variable

1. $x = at^2$, $y = 2at$

Solution:

By differentiating x w.r.t. t

$$\frac{dx}{dt} = \frac{d(at^2)}{dt}$$

$= 2at$

By differentiating y w.r.t. t

$$\frac{dy}{dt} = \frac{d(2at)}{dt}$$

$= 2a$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$$

θ is the 3rd variable

2. x = a cos θ, y = b sin θ

Solution:

By differentiating x w.r.t. **θ**

TRIGO. AGAIN

$$\frac{dx}{d\Theta} = -a sin\Theta \qquad \frac{dy}{d\Theta} = b cos\Theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{-a\sin\theta}{b\cos\theta} = \frac{-a}{b}\tan\theta$$

θ is the 3^{rd} variable

$x = a(1 - \cos\theta),\ y = a(\theta + \sin\theta)$

Solution:

By differentiating x w.r.t. θ

$$\frac{dx}{d\theta} = \frac{da(1 - \cos\theta)}{d\theta}$$

So we get

$$\frac{dx}{d\theta} = a\sin\theta.$$

By differentiating y w.r.t. θ

$$\frac{dy}{d\theta} = \frac{da(\theta + \sin\theta)}{d\theta}$$

So we get

$$\frac{dy}{d\theta} = a \times (1 + \cos\theta)$$

Trigo. identities

M. W. F

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$= \frac{a(1+\cos\theta)}{a(\sin\theta)}$$

$$= \frac{2\cos^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)}$$

So we get

$= \cot(\theta/2)$

t is the 3 rd variable

$x = (\log t + \cos t), y = (e^t + \sin t)$

Solution:

By differentiating x w.r.t. t

$$\frac{dx}{dt} = \frac{d(\log t + \cos t)}{dt}$$

So we get

$$= \frac{1}{t} - \sin t.$$

By differentiating y w.r.t. t

$$\frac{dy}{dt} = \frac{d(e^t + \sin t)}{dt} = e^t + \cos t$$

$$\frac{dy}{dx} = \frac{e^t + \cos t}{\frac{1}{t} - \sin t}$$

$$= \frac{t(e^t + \cos t)}{1 - t \sin t}$$

Find $\frac{dy}{dx}$

$x = \cos\theta + \cos 2\theta,\ y = \sin\theta + \sin 2\theta$

Solution:

By differentiating x w.r.t. θ

$$\frac{dx}{d\theta} = \frac{d(\cos\theta + \cos 2\theta)}{d\theta}$$

So we get

$= -\sin\theta - 2\sin 2\theta$

$$\frac{dy}{d\theta} = \frac{d(\sin\theta + \sin 2\theta)}{d\theta}$$

So we get

$= \cos\theta + \cos 2\theta \times 2$

By substituting the values we get

$$\frac{dy}{dx} = \frac{\cos\theta + 2\cos 2\theta}{-(\sin\theta + 2\sin 2\theta)}$$

Chain rule

Find $\frac{dy}{dx}$

Key point
Product rule

x = a (cos θ + θ sin θ), y = a (sin θ − θ cos θ).

Solution:

By differentiating x w.r.t. θ

$$\frac{dx}{d\theta} = \frac{d\ a(\cos\theta + \theta\sin\theta)}{d\theta}$$

We know that

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

So we get

= a(-sinθ +θ cosθ +sinθ)

= a θ cos θ

By differentiating y w.r.t. θ

$$\frac{dy}{d\theta} = \frac{d\ a(\sin\theta - \theta\cos\theta)}{d\theta}$$

$$\frac{dy}{d\theta} = \frac{d\ a(\sin\theta - \theta\cos\theta)}{d\theta}$$

We know that

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

So we get

= a(cosθ - (-θ sinθ + cosθ))

= a θ sin θ

By substituting the values we get

$$\frac{dy}{dx} = \frac{a\times\theta\sin\theta}{a\times\theta\cos\theta}$$

So we get

= tan θ